Systematic search for functional programs

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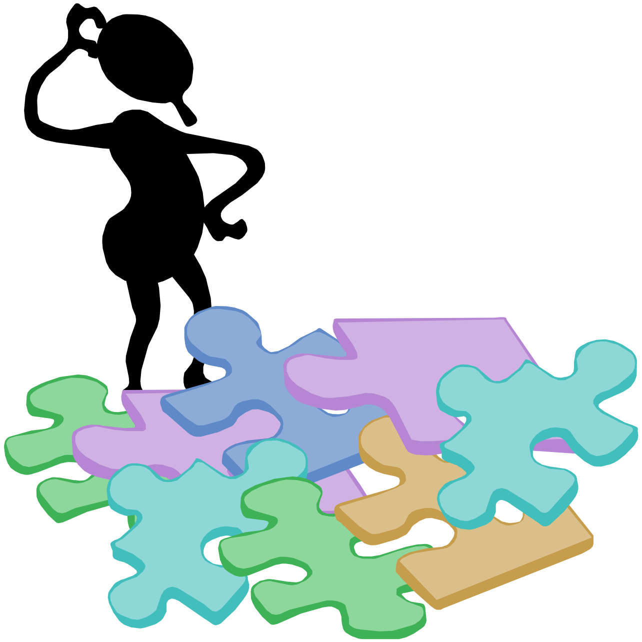
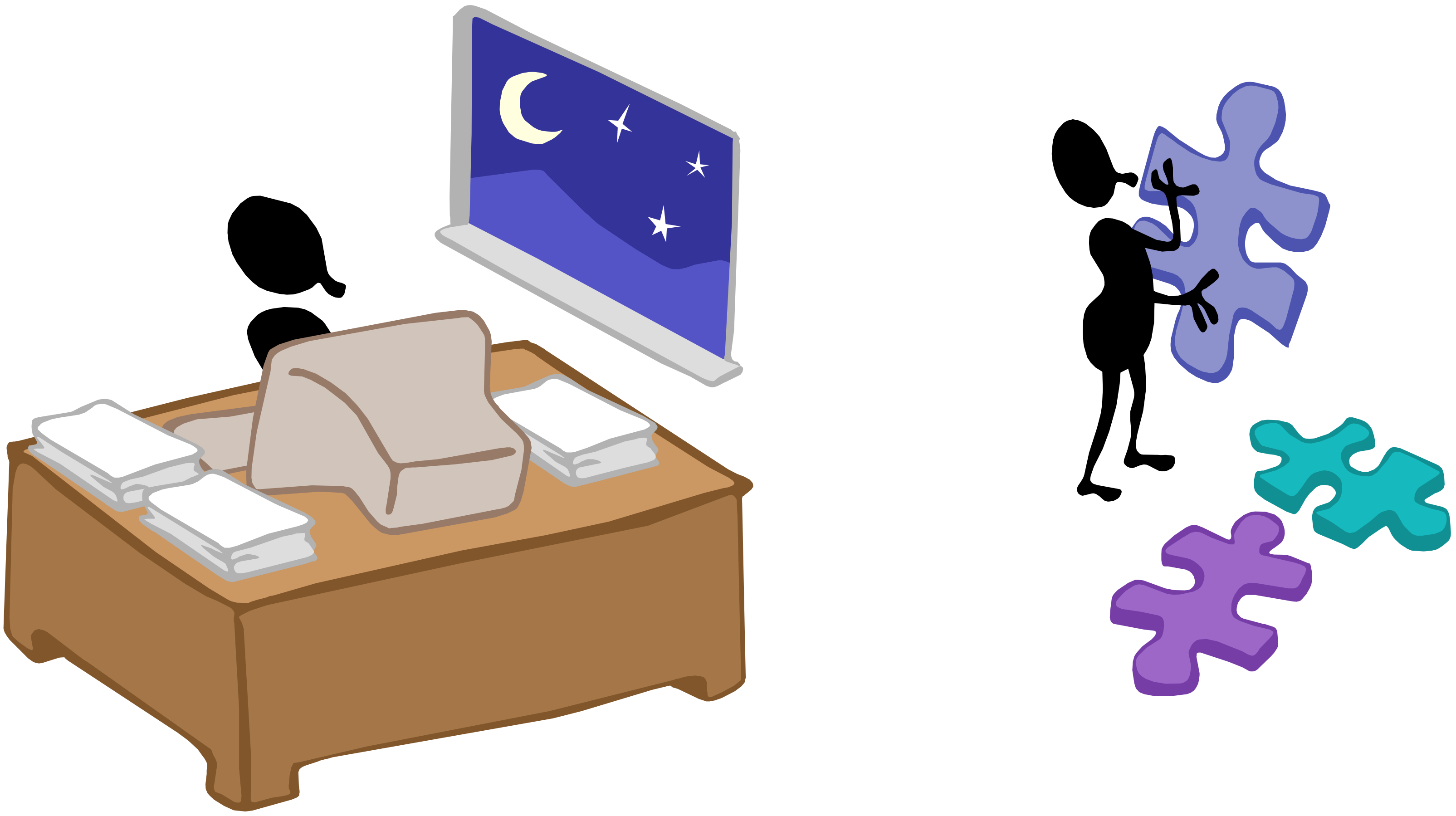
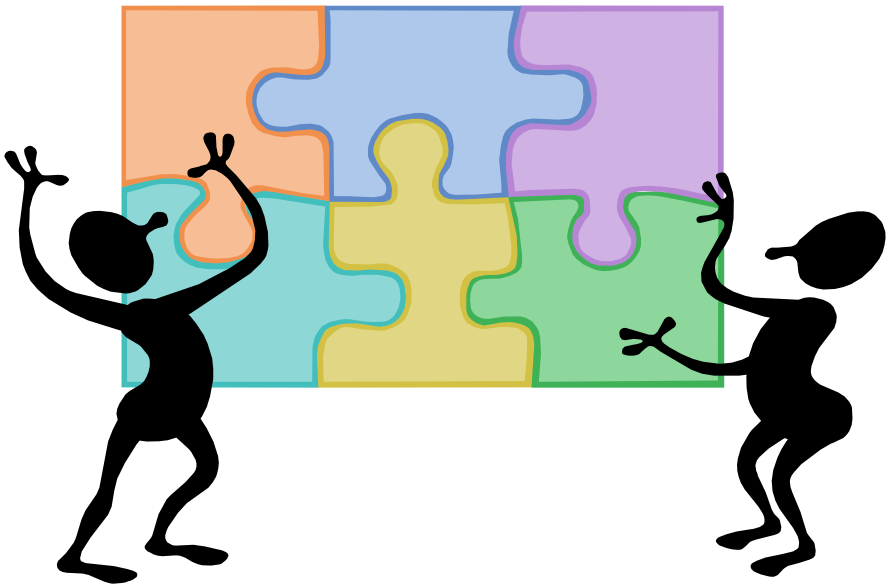
Exploiting strong typing



Exploiting strong typing



Exploiting strong typing



Inductive synthesis of functional programs

Inductive synthesis of functional programs (Inductive functional programming)
automatic generation of **functional programs** from **examples/properties**.

examples (I/O pair)
{("dsl",3), ("ak",2)}

or

properties (predicate)
f s.t. f "dsl" == 3 &&
f "ak" == 2

→

functional program

f "" = 0
f (c:s) = 1 + f s

* Properties can be more flexible than examples.

The implemented system

Please watch the demo….

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Human programmers usually:

could be automated

Write a function → review it → test it

You should still do that!

Outline

- **Introduction**

  motivation, policy, & conventional approaches

- **The implemented system**

  rough specification & detailed implementation

- **Experiments**

  results on some easy problems

- **Conclusions**

Placement

- “Deductive” synthesis from complete spec.
- “Inductive” synthesis from incomplete spec.
  - by generating & folding computational traces from examples
  - search-based, or by trial and error
    - Genetic Programming (GP)
    - The proposed method

related work(1): Synthesis via computational traces

- Step 1: Generate computational traces
  - by constraints on examples (Summers 1977)
  - by hand (a.k.a. programming by demonstration (PBD))
  - by a universal planner (Schmid 2001)
  - by genetic programming (Schmid 2004)

  by search
- Step 2: Fold the traces into a recursive program
  - by pattern matching

Drawback: **unfolded traces are longer than programs**

search space bloats much earlier

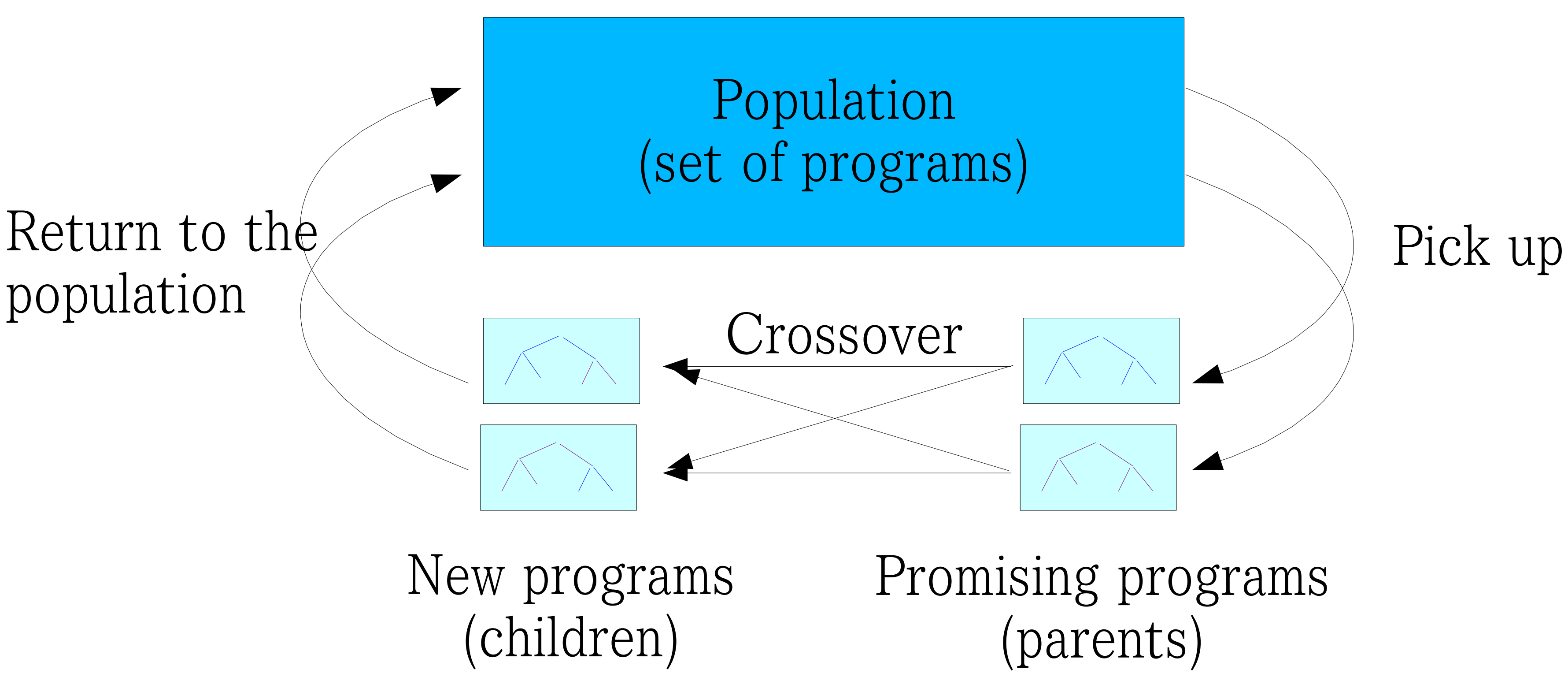
→ (than direct search methods like GP)

related work(2): *Genetic programming* (1/3)

assumption:

"useful programs consist of useful subexpressions"

Search by recombination of subexpressions



related work(2): *Genetic programming* (2/3)

GP applied to inductive algorithm synthesis

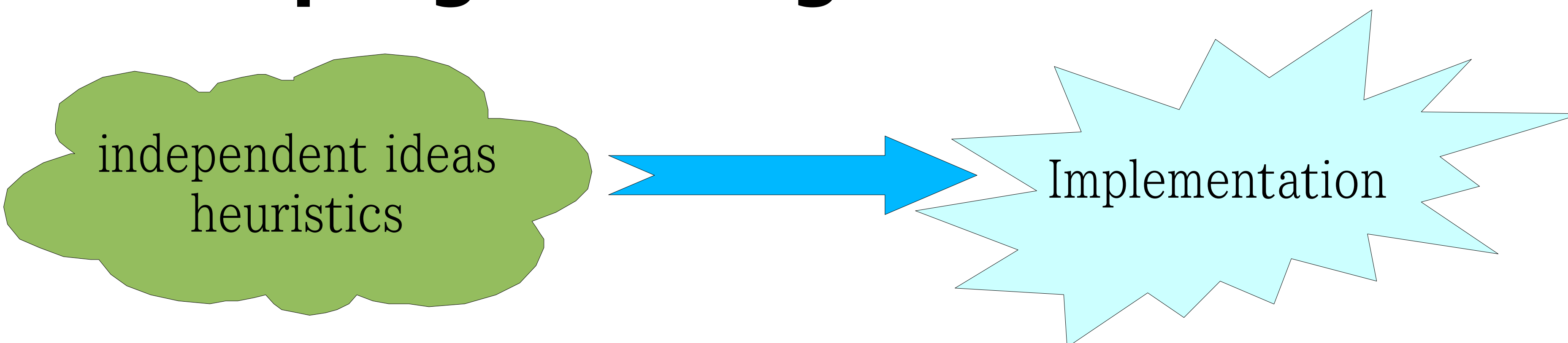
our interest:
synthesis of **reusable** (often **typeful** and **recursive**) algorithms

NB: typical use of GP: typeless CSP and function approx.

- ADATE (Olsson 1995)
  - monomorphic first-order type system
  - requires a file with tens of lines written for each synthesis
  - interesting results reported, but not reproducible
- PolyGP (Yu 1998)
  - polymorphic higher-order type system
  - requires a file with tens of lines written for each synthesis
  - exhaustive search is faster (Katayama 2004)

related work(2):
Genetic programming (3/3)

Genetic programming research



- different scientists use different settings
  “100 researchers, 100 genetic algorithms
- no comparison with non-heuristic approaches

It is often unclear if/which heuristic worked.

Motivation

1) First we should investigate systematic search.

- How can we optimize its implementation?
- How efficient is it? Is it slower/faster than GP?
- etc.

2) Heuristics (incl‘ GP) should be added based on the above knowledge.

- better modularity
- steady improvement

Policy (1)

- exploration of **exhaustive breadth-first search**
  - construct a basis on which to build heuristic approaches like GP,
  - control the exponential bloat in the search space.

to be more concrete,

- Limit to **(grammatically & type)-correct** expressions (of course!)
- Avoid multiple-count of **equivalent expressions**

  using known **reducible patterns** and (maybe) **transformation rules**

  e.g. *do not try both* `foldr foo bah []` *and* `bah`

  do not try both `(¥x -> foo x) bah` *and* `foo bah`
- **Optimize** the implementation --- **memoization**, etc.

Policy (2)

- **ease of use**

  at least it should be easier than writing the programs directly

  - <u>just writing a boolean function as predicate</u> invokes the search
  - use <u>general-purposed component combinator set</u>, not tailor-made function set for each synthesis

The implemented system

- **Introduction**
  motivation, policy, & conventional approaches
- **The implemented system**
  rough specification & detailed implementation
- **Experiments**
  compared with polymorphic GP
- **Conclusions**

Specification

keyboard

- property,
  e.g., \f. f “slkd”==4
- type (optional)
  e.g. [a] ──► Int

* The type is inferred when omitted.

component library file

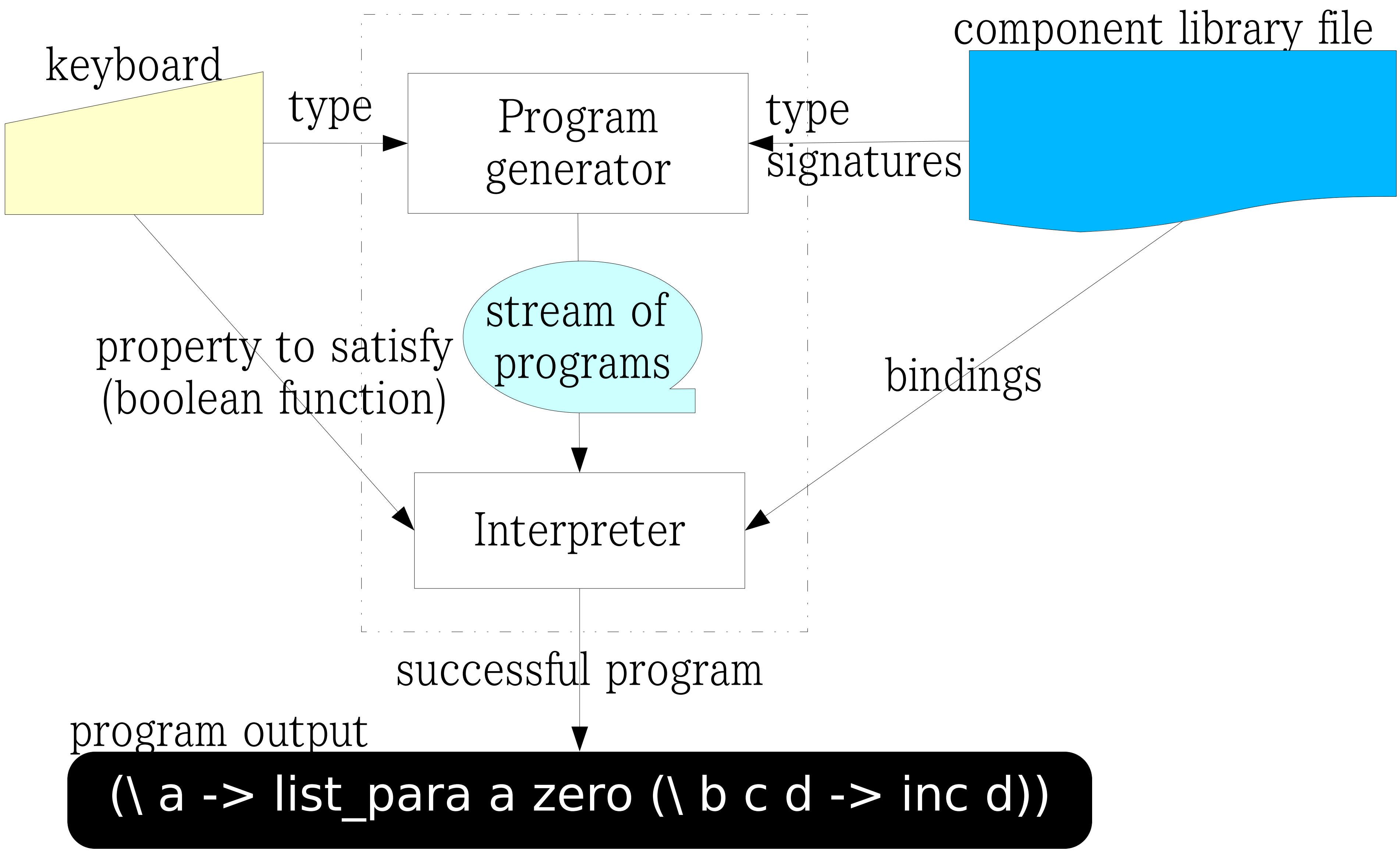
```
zero :: Int
zero = 0
inc   :: Int -> Int
inc   = \ x -> x+1
nat_para :: Int -> a->(Int->a->a)->a
nat_para = \ i x f ->
               if i==0 then x else
               f (i-1) (nat_para (i-1) x f)
     ...
```

synthesizer

program output

```
(\ a -> list_para a zero (\ b c d -> inc d))
```

Internal structure



Language

The language: Haskell subset

- Hindley-Milner type system

  without functions in containers, such as [a->b], (a,b->c), ...(for efficiency reasons)

- Frontend … usual lambda calculus

  Backend … de Bruijn lambda calculus

Katayama (2004) used typed *SIBC* combinators, but

- Such combinatory expressions are redundant;
- Search tree branches a lot in the shallow stages;
- The complexity of involved types bloats.

Implementation

Story:

- Preparation
  - Monad for breadth-first search
  - Monad transformer for type inference
- Arity known cases **where functions never return type vars**
- Arity unknown cases

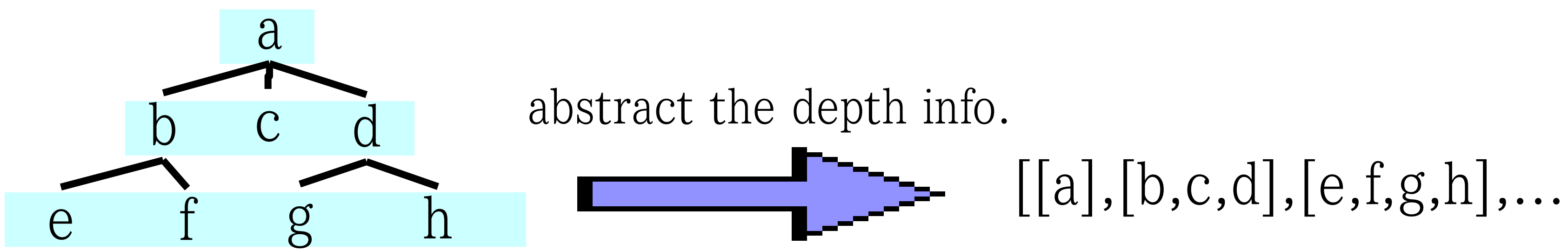
  where there can be a function of the form “forall a. Foo → Bar → … → a”
- Memoization **& devices for making keys hit often**

Monad for breadth-first search

- Spivey (2000)'s monad:

  Matrix a = Stream (Bag a)   (actually defined as [[a]])

  1st bag for depth 0, 2nd for depth 1, … of the search tree



  $\times$ **for combinations and $+$ for alternatives (defined as usual)**

  Eases implementation a lot but gobbles the heap a lot

- Recomputing variant:

  Recomp a = $\underbrace{\text{Int}}_{\text{depth}}$ $\rightarrow$ Bag a suppresses heap consumption

Monad trans. for type inference

- Typical implementation:

  TI a = Subst → Int → Maybe (a, Subst, Int)

  states

  Subst: "current" substitution

  Int: fresh variable ID

- **Monad transformer** considering **alternative states:**

  TI Recomp a

  where TI m a = Subst → Int → m (a, Subst, Int)

Naïve implementation (arity known)

```
data Expression = Lambda Expression          -- lambda abstraction
                | X        Int               -- de Bruijn variable
                | Expression :$ Expression   -- function application

uniExprs :: [(Expression,Type)] -> [Type] -> Type -> TI Recomp Expression
uniExprs prims avails (t0:->t1) = do result <- uniExprs prims (t0:avails) t1
                                     return (Lambda result)
uniExprs prims avails reqret
    = do (expr,typ) <- msum $ map return $ (zip (map X [0..]) avails ++ prims)
         typ'    <- freshVariablesForForalls typ
         setMGU reqret (returnTypeOf typ')    -- setMGU :: Type -> Type -> TI m ()
         spine avails typ' expr
   where spine :: Type -> Expression -> TI Recomp Expression
         spine (t:->ts) fun                  -- (:->) represents function type constructor
           = lift delay (do subst <- getSubst
                            arg <- uniExprs prims (map (apply subst) avails) (apply subst t)
                            spine ts (fun :$ arg) )
         spine _avs   _t      fun = return fun
```

***simplified to improve presentation rather than the efficiency**
e.g. Expressions with the same type are actually put together to [Expression]

Primitives returning type variables (arity unknown)

$\forall a. T_1 \rightarrow T_2 \rightarrow a$ can be specialized

$T_1' \rightarrow T_2' \rightarrow R, T_1'' \rightarrow T_2'' \rightarrow (A_1 \rightarrow R), T_1''' \rightarrow T_2''' \rightarrow (A_1 \rightarrow A_2 \rightarrow R), \ldots$

($T_1', T_2', T_1'', T_2'', T_1''', T_2''', \ldots$ are specialization of T_1 and T_2)

➡ **Arity undecidable**

- solution 1: generate all alternatives

  (a $\mapsto$ R) case `*mplus*` (a $\mapsto$ b -> R) case `*mplus*` (a $\mapsto$ b -> c -> R…

  problem: create lots of equivalent programs

- solution 2(current): introduce new constructor representing direct product of type variables:

  b, (b,c), (b,c,d), …

Memoization

The same simple type is requested many times

Memoize $\quad Type \to [Type] \to TI\ Recomp\ [Expression]$

(or* $\underline{Type} \to \underline{[Type]} \to \underline{Int} \to [([Expression], Subst, Int)]$)

request type

Types of available variables

depth

Queries often hit.

This list becomes long.

* **Exactly,** $\mathrm{Type} \to [\mathrm{Type}] \to \mathrm{Subst} \to \mathrm{Int} \to \mathrm{Int} \to [(\mathrm{Expression}, \mathrm{Subst}, \mathrm{Int})]$

but you could apply the substitution and rename the type variables to normalize its numbering.

Reorganizing (idea)

Point:

Only the *set* of available *types* matters

$\times$ list

e.g. for $x_0, x_1 :: Char$

$$\ldots x_0 \ldots x_0 \ldots \text{ is type correct} \Leftrightarrow \ldots x_0 \ldots x_1 \ldots \text{ is type correct}$$
$$\Leftrightarrow \ldots x_1 \ldots x_0 \ldots \text{ is type correct}$$
$$\Leftrightarrow \ldots x_1 \ldots x_1 \ldots \text{ is type correct}$$

$available = \{x_0, x_1 :: Char\}$ case and
$available = \{x_0 :: Char\}$ case can share the same memo entry*

* *post processing required*

Reorganizing (implementation)

Available variables: $x_0 :: Char, x_1 :: Bool, x_2 :: Char$

preprocess (sort & nub)

Memo entry: $x_0 :: Char, x_1 :: Bool$

Retriever map: $\{0 \mapsto [0,2], 1 \mapsto [1]\}$

Memo function wrapper

1. sort the argument types and assign **one variable name** for **one type**
2. invoke memo function
3. generate all cases by replacing variables (using retriever map).

Avoiding reducible expressions

- Optimization rules suggest redundancy in the program space.
- Because expressions with the same type are put together, rules that cannot detected from type info. might not help a lot.
- Current rule to identify reducible points:
  the strict argument of consumer functions
  (like case, cata, para) must not be constant
  (i.e. must include a free variable)

(Note: quite limited)

Experiments

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Problems

Problems from the previous work (Katayama 2004) for comparison

- nth :: Int → [a] → a
  nth 5 “widjfgwi” == 'f' &&
  nth 1 “wddidjfgwi” == 'w'
- map :: (b → a) → [b] → [a]
  map (=='c') “stock” == [False,False,False,True,False] &&
  map (=='e') “peeped” == [False,True,True,False,True,False]
- length :: [a] → Int
  length “hageho” == 6 && length “hoge” == 4

The component combinators

Previous work

- **S**, **K**, **I**, **B**, and **C**
- constructors
- curried paramorphisms
- head, tail and pred

New algorithm

- constructors
- curried paramorphisms
- head, tail and pred

Results

Evaluation of proposed method:

Computation time (sec.)	nth	map	length
Old algorithm (real)	5.3	2.2	0.03
(user)	5.1	2.2	0.02
New algorithm (real)	0.8	1.9	0.03
(user)	0.6	1.2	0.02

- Improved for the problems used in the previous work.
- Greater programs (sized more than 12) still require more than a minute or cannot be synthesized.

Discussion towards a usable system:
# of equivalent programs

The results so far are happened-to-work toy examples!

Synthesis of *take::Int->[a]->[a]* requires tens of seconds.

.... How much it might **potentially** be optimized?

→ Do (very) light-weighted random testing

EXPERIMENT TO GIVE SOME ESTIMATE:

for each *f :: Int -> [a] -> [a]* in the generated expressions,

- compute *(f 2 "12332", f 0 "56789", f 2 "k", f 0 "")* > file
- compare wc file and sort file | uniq | wc

file	... 251940 lines
sort file | uniq	... 514 lines

LOTS OF ROOM FOR IMPROVEMENT

Conclusions

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Summary

- investigated breadth-first exhaustive search for de Bruijn expressions
- easy problems sized around ten can be solved within seconds
- Still room for efficiency improvement

Future work

- Controlling exponential bloat

  Solution: **remove more equivalent programs**

  or lower the priorities of seem-to-be-equivalent programs

  - transformation rules
  - use lightweighted random testing for small expressions
- Type classes: **dealing with contexts**
  - contexts "generic in nature" e.g. Eq, Show, Read, etc. … just ignore.
  - other "ad hoc" classes like Num … needs implementation.
- Heuristics:

  e.g. to prioritize subexpressions of expressions
  that returned the correct outputs to some of the inputs